Problem §7.1 - 27(a): Find the probability of selecting exactly one of the correct six integers in a lottery, where the order in which these integers are selected does not matter, from the positive integers not exceeding 40.

Problem §7.1 - 36: Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled? Show your work.

Problem §7.2 - 8(a,c,d): What is the probability of these events when we randomly select a permutation of $\{1, 2, \ldots, n\}$ where $n \geq 4$?

(a) 1 precedes 2.

(c) 1 immediately precedes 2.

(d) n precedes 1 and $n - 1$ precedes 2.

Problem §7.2 - 18:

(a) What is the probability that two people chosen at random were born on the same day of the week?

(b) What is the probability that in a group of n people chosen at random, there are at least two born on the same day of the week?

(c) How many people chosen at random are needed to make the probability greater than 1/2 that there are at least two people born on the same day of the week?

Problem §7.2 - 24: What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

Problem §7.2 - 30: Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if

(a) a 0 bit and a 1 bit are equally likely.

(b) the probability that a bit is 1 is 0.6.

(c) the probability that the ith bit is a 1 is $1/2^i$ for $i = 1, 2, 3, \ldots, 10$.

Problem §7.2 - 34: Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

(a) the probability of no successes.

(b) the probability of at least one success.

(c) the probability of at most one success.

(d) the probability of at least two successes.

Problem §7.2 - 36: Use mathematical induction to prove that if $E_1, E_2, \ldots, E_n$ is a sequence of n pairwise disjoint events in a sample space S, where n is a positive integer, then

$$p\left(\cup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} p(E_i).$$

Problem §7.3 - 2: Suppose that E and F are events in a sample space and $p(E) = 2/3$, $p(F) = 3/4$, and $p(F \mid E) = 5/8$. Find $p(E \mid F)$.

Problem §7.3 - 8: Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 99.9% of people with the disease test positive and only 0.02% without the disease test positive.

(a) What is the probability that someone who tests positive has the genetic disease?

(b) What is the probability that someone who tests negative does not have the disease?