Problem §6.3 - 16: How many subsets with an odd number of elements does a set with 10 elements have?

Problem §6.3 - 20: How many bit strings of length 10 have

(a) exactly three “0”s?

(b) more “0”s than “1”s?

(c) at least seven “1”s?

(d) at least three “1”s?

Problem §6.3 - 24: How many ways are there for ten women and six men to stand in a line so that no two men stand next to each other?

Problem §6.3 - 42: Find a formula for the number of ways to seat r of n people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.

Problem §6.4 - 10: Give a formula for the coefficient of x^k in the expansion of $(x + 1/x)^{100}$, where k is an integer.

Problem §6.4 - 12: The row of Pascal’s triangle containing the binomia coefficients $\binom{10}{k}$, for $0 \leq k \leq 10$, is:

$$1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1$$

Use Pascal’s identity to produce the row immediately following this row in Pascal’s triangle.

Problem §6.4 - 22: Prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever n, r, and k are non-negative integers with $r \leq n$ and $k \leq r$,

(a) using a combinatorial argument.

(b) using an argument based on the formula for the number of r-combinations of a set with n elements.

Problem §6.4 - 27(a): Prove the **hockeystick identity**

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$$

whenever n and r are positive integers, using a combinatorial argument.

Problem §6.5 - 10(a,c,d): A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

(a) a dozen croissants?

(c) two dozen croissants with at least two of each kind?

(d) two dozen croissants with no more than two broccoli croissants?

Problem §6.5 - 20: How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 11,$$

where x_1, x_2, and x_3 are non-negative integers?

Problem §6.5 - 26: How many positive integers less than $1,000,000$ have exactly one digit equal to 9 and have a sum of digits equal to 13?

Problem §6.5 - 46: A shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen?

Problem §7.1 - 10: What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?

Problem §7.1 - 12: What is the probability that a five-card poker hand contains exactly one ace?

Problem §7.1 - 14: What is the probability that a five-card poker hand contains cards of five different kinds?

Problem §7.1 - 16: What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?