**Problem §5.2: 8:** Suppose that a store offers gift certificates in denominations of 25 and 40 dollars. Determine the possible total amounts you can form using these gift certificates. Prove your answer using strong induction.

**Problem §5.2: 10:** Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. The entire bar, a smaller rectangular piece of the bar, can be broken along on a vertical or horizontal line separating the squares. Assuming that only one piece can be broken at a time, determine how many breaks you must successively make to break the bar into n separate squares. Use strong induction to prove your answer.

**Problem §6.1: 8:** How many different three-letter initials with none of the letters repeated can people have?

**Problem §6.1: 14:** How many bit strings of length n, where n is a positive integer, start and end with 1s?

**Problem §6.1: 16:** How many strings are there of four lowercase letters that have the letter *x* in them?

Problem §6.1: 26: How many strings of four decimal digits

- (a) do not contain the same digit twice?
- (b) end with an even digit?
- (c) have exactly three digits that are 9s?

**Problem §6.1: 30:** How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

**Problem §6.1: 36:** How many functions are there from the set  $\{1, 2, ..., n\}$ , where n is a positive integer, to the set  $\{0, 1\}$ ?

**Problem §6.1: 37:** How many functions are there from the set  $\{1, 2, ..., n\}$ , where n is a positive integer, to the set  $\{0, 1\}$ 

- (a) that are one-to-one?
- (b) that assign 0 to both 1 and n?
- (c) that assign 1 to exactly one of the positive integers less than n?

**Problem §6.1: 40:** How many subsets of a set with 100 elements have more than one element?

**Problem §6.1: 44:** How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

**Problem §6.2: 2:** Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

**Problem §6.2: 6:** Let d be a positive integer. Show that among any group of d + 1 (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by d.

**Problem §6.2: 8:** Show that if f is a function from S to T, where S and T are finite sets with |S| > |T|, then there are elements  $s_1$  and  $s_2$  in S such that  $f(s_1) = f(s_2)$ , or in other words, f is not one-to-one.

## Problem §6.2: 14:

- (a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.
- (b) Is the conclusion in (a) true if six integers are selected rather than seven?