**Problem §3.2: 2(a,b,e,f):** Determine whether each of these functions is $O(x^2)$:

(a) $f(x) = 17x + 11$.

(b) $f(x) = x^2 + 1000$.

(e) $f(x) = 2^x$.

(f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$.

**Problem §3.2: 8:** Find the least integer $n$ such that $f(x)$ is $O(x^n)$ for each of these functions.

(a) $f(x) = 2x^2 + x^3 \log x$.

(b) $f(x) = 3x^5 + (\log x)^4$.

(c) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$.

(d) $f(x) = (x^3 + 5 \log x)/(x^4 + 1)$.

**Problem §3.2: 17:** Suppose that $f(x)$, $g(x)$, and $h(x)$ are functions such that $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Show that $f(x)$ is $O(h(x))$.

**Problem §3.2: 26:** Give a big-$O$ estimate for each of these functions. For the function $g$ in your estimate $f(x)$ is $O(g(x))$, use a simple function $g$ of the smallest order.

(a) $f(x) = (n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)$.

(b) $f(x) = (2^n + n^2)(n^3 + 3^n)$.

(c) $f(x) = (n^n + n2^n + 5^n)(n! + 5^n)$.

**Problem §3.2: 28(a,b,c,d):** Determine whether each of the following functions is $\Omega(x)$ and whether it is $\Theta(x)$.

(a) $f(x) = 10$.

(b) $f(x) = 3x + 7$.

(c) $f(x) = x^2 + x + 1$.

(d) $f(x) = 5 \log x$.

**Problem Extra:** Explain what it means for a function to be

(a) $O(1)$.

(b) $\Omega(1)$.

(c) $\Theta(1)$.

**Problem §5.1: 4:** Let $P(n)$ be the statement that $1^3 + 2^3 + \cdots + n^3 = (n(n+1)/2)^2$ for the positive integer $n$.

(a) What is the statement $P(1)$?

(b) Show that $P(1)$ is true, completing the basis step of the proof.

(c) What is the inductive hypothesis?

(d) What do you need to prove in the inductive step?

(e) Complete the inductive step, identifying where you use the inductive hypothesis.

(f) Explain why these steps show that this formula is true whenever $n$ is a positive integer.

**Problem §5.1: 6:** Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$ whenever $n$ is a positive integer.

**Problem §5.1: 8:** Prove that $2 - 2 \cdot 7 + 2 \cdot 7^2 - \cdots + 2(-7)^n = (1 - (-7)^{n+1})/4$ whenever $n$ is a nonnegative integer.

**Problem §5.1: 20:** Prove that $3^n < n!$ if $n$ is an integer greater than 6.

**Problem §5.1: 34:** Prove that 6 divides $n^3 - n$ whenever $n$ is a nonnegative integer.

**Problem §5.1: 49:** What is wrong with this "proof" that all horses are the same color?

Let $P(n)$ be the proposition that all the horses in a set of $n$ horses are the same color.

*Basis Step:* Clearly, $P(1)$ is true.

*Inductive Step:* Assume that $P(k)$ is true, so that all the horses in any set of $k$ horses are the same color. Consider any $k+1$ horses: number these horses as $1, 2, 3, \ldots, k, k+1$. Now the first $k$ of these horses all must have the same color. Because the set of the first $k$ horses and the set of the last $k$ horses overlap, all $k+1$ must be the same color. This shows that $P(k+1)$ is true and finishes the proof by induction.

**Problem §5.1: 51:** What is wrong with this "proof"?

"*Theorem*": For every positive integer $n$, if $x$ and $y$ are positive integers with $\max(x, y) = n$, then $x = y$.

*Basis Step:* Suppose that $n = 1$. If $\max(x, y) = 1$ and $x$ and $y$ are positive integers, we have $x = 1$ and $y = 1$.

*Inductive Step:* Let $k$ be a positive integer. Assume that whenever $\max(x, y) = k$ and $x$ and $y$ are positive integers, then $x = y$. Now let $\max(x, y) = k + 1$, where $x$ and $y$ are positive integers. Then $\max(x - 1, y - 1) = k$, so by the inductive hypothesis $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step.