## Homework 2

Problem §2.3: 2: Determine whether f is a function from Z to R if
(a) f(n) = ±n
(b) f(n) = √n<sup>2</sup> + 1

- (c)  $f(n) = \frac{1}{n^2 4}$

**Problem §2.3: 12:** Determine whether each of these functions from  $\mathbb{Z}$  to  $\mathbb{Z}$  is one-to-one.

- (a) f(n) = n − 1.
  (b) f(n) = n<sup>2</sup> + 1.
- (0) j(10) 10 1
- (c)  $f(n) = n^3$ .
- (d)  $f(n) = \lceil n/2 \rceil$ .

**Problem §2.3:** 14(a,b,c,d): Determine whether  $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$  is onto if

- (a) f(m,n) = 2m n.
- (b)  $f(m,n) = m^2 n^2$ .
- (c) f(m,n) = m + n + 1.
- (d) f(m,n) = |m| |n|.

**Problem §2.3: 20:** Give an example of a function from  $\mathbb{N}$  to  $\mathbb{N}$  that is

- (a) one-to-one but not onto.
- (b) onto but not one-to-one.
- (c) both onto and one-to-one (but not the identity function).
- (d) neither one-to-one nor onto.

**Problem §2.3: 22(a,b):** Determine whether each of these functions is a bijection from  $\mathbb{R}$  to  $\mathbb{R}$ .

- (a) f(x) = -3x + 4.
- (b)  $f(x) = -3x^2 + 7$ .

**Problem §2.3: 36:** Find  $f \circ g$  and  $g \circ f$  where  $f(x) = x^2 + 1$  and g(x) = x + 2 are functions from  $\mathbb{R}$  to  $\mathbb{R}$ .

**Problem §2.3: 39:** Show that the function f(x) = ax + b from  $\mathbb{R}$  to  $\mathbb{R}$  is invertible, where a and b are constants, with  $a \neq 0$ , and find the inverse of f.

Homework 2

**Problem §2.3: 40(a):** Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that  $f(S \cup T) = f(S) \cup f(T)$ .

**Problem §2.3:** 44(b): Let f be a function from A to B. Let S and T be subsets of B. Show that  $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$ .

**Problem §3.1: 2:** Determine which characteristics of an algorithm described in the text the following procedures have and which they lack.

(a) procedure double(n: positive integer) while n > 0n := 2n (b) procedure divide(n: positive integer) while  $n \ge 0$ m : = 1/nn := n-1 (c) procedure sum(n: positive integer) sum := 0while i < 10sum := sum + i (d) procedure choose(a,b: integers) x := either a or b

**Problem §3.1: 24:** Describe an algorithm that determines whether a function from a finite set to another finite set is one-to-one.

**Problem §3.1: 52(a,d):** Use the greedy algorithm to make change using quarters, dimes, nickels, and pennies for

(a) 87 cents.

(d) 33 cents.

**Problem §3.1: 54(a,d):** Use the greedy algorithm to make change using quarters, dimes, and pennies (but no nickels) for

(a) 87 cents.

(d) 33 cents.